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$B_s \to \mu^+\mu^-$ and the upward-going muon flux from the WIMP annihilation in the sun or the earth

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Abstract: We consider the upward-going muon flux due to the WIMP annihilations in the cores of the sun and the earth, including the upper bound on the branching ratio for $B_s \to \mu^+\mu^-$ decay. We find that the constraint from $B_s \to \mu^+\mu^-$ is very strong in most parameter space, and exclude the supergravity parameter space regions where the expected upward-going muon fluxes are within the expected reach of IceCube.

Keywords: B-Physics, Supersymmetry Phenomenology.

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Contents

1. Introduction

There is now compelling evidence for a non-baryonic cold dark matter (DM) component in the universe [1]. In the Minimal Supersymmetric Standard Model (MSSM) with R parity, the lightest supersymmetric particle (LSP) is stable and becomes a good candidate for cold dark matter in the universe [2]. The LSP is often the lightest neutralino which is the admixture of Bino, Wino and Higgsinos in the MSSM. In this case, the neutralino DM in our galactic halo might be detected via its elastic scattering with terrestrial nuclear targets [3, 4]. In fact, the DAMA Collaboration [5] even claimed an evidence for DM. However, the CDMS II experiment [6] has reported the upper limit on the DM scattering cross section, which is not compatible with the results of the DAMA experiment. There are several experiments going on searching for the DM scattering at the level of $\sigma_{\chi p}^{\rm SI} \sim 10^{-7}$ pb or less. In the most widely studied minimal supergravity (mSUGRA) scenario (or the constrained MSSM), the spin-independent neutralino-proton scattering cross section $\sigma_{\chi p}$ turns out very small ($\lesssim 10^{-8}$ pb). However there is no solid theoretical rationale for the minimal supergravity scenario, and it is important to calculate the possible maximal values for $\sigma_{\chi p}$ in *general* supergravity scenarios beyond the mSUGRA scenario. And it is very important to impose all the relevant constraints from various experiments in order not to overestimate the cross section. Some important constraints include the lower bounds on the Higgs and SUSY particle masses, $B \to X_s \gamma$ branching ratio, the muon $(g-2)_\mu$, etc.. One may also take some theoretical consideration on the absence of the color-charge breaking minima or the directions unbounded from below, etc.

In a previous work [7], we pointed out that there is a strong correlation between the spin independent neutralino-proton scattering cross section $\sigma_{\chi p}$ and the branching ratio for $B_s \to \mu^+\mu^-$ decay [8 – 12] within mSUGRA and its extensions. The origin of this correlation resides in the dependence of both observables on $\tan\beta$ and the neutral Higgs boson masses; both observables increase for large $\tan\beta$ and low Higgs masses. In particular, we have shown that the current upper limit on $B(B_s \to \mu^+\mu^-)$ excludes substantial parameter space where the DM scattering cross section is within the CDMS sensitivity region [7] (see also [13] for a detailed analysis).

In this work, we extend our previous study to the indirect detection of neutralino DM with neutrino telescope through upward-going muon flux, and its correlation with $B(B_s \to \mu^+\mu^-)$. The energetic neutrino(-induced muon) flux from neutralino DM annihilation in the sun and the earth is one of the promising signals in the indirect detection of neutralino DM [14]. Neutralino DM particles in the halo can be captured by the sun or by the earth, when their velocities drop below escape velocities via their elastic scattering with matter in the sun or earth. Then they will accumulate in the core of the sun and the earth and will eventually annihilate into ordinary SM particles. Among the annihilation products, neutrinos can pass through the sun and the earth, and then could be detected in neutrino telescopes through their conversion to muons via charged-current scattering with nuclei near the detectors. Baksan [15], MACRO [16], Super-K [17] and AMANDA [18] released upper limits on the upward-going muon flux. There are also planned or proposed neutrino telescopes such as ANTARES [19], IceCube [20] and NESTOR [21] etc..

An important point of the indirect detection of neutralino DM with neutrino telescopes is that the neutrino flux strongly depends on the capture rate of neutralino by the sun or the earth, which in turn depends on neutralino-nucleon scattering cross sections. Therefore we expect some correlation between the neutrino flux and $B(B_s \to \mu^+\mu^-)$, which is similar to the strong correlation between $\sigma_{\chi p}$ and $B(B_s \to \mu^+\mu^-)$ as discussed in ref. [7]. Indeed, we will show that the current upper limit of $B(B_s \to \mu^+\mu^-) < 4.1 \times 10^{-7}$ (90 % CL) [22, 23] puts strong constraints on the upward-going muon flux in the supersymmetric models which give rather large spin-independent neutralino-proton scattering cross section.

This paper is organized as following. In section 2, we give a brief review on the indirect detection of the DM through the upward-going muon flux. In section 3, we consider the upward-going muon fluxes in some supergravity scenarios and illuminate our point that $B_s \to \mu^+\mu^-$ branching ratio plays an important role. In section 4, we summarize the results.

2. Indirect detection through the upward-going muon flux

As we mentioned in the introduction, the observation of energetic neutrinos from the sun and/or the earth would provide convincing evidence of the existence of neutralino dark matter in galactic halo [4]. The flux of energetic neutrinos from neutralino annihilation in the sun or the earth is proportional to the rate of neutralino annihilation in the sun or in the earth and the energy spectrum of neutrinos from the annihilation. The time evolution of the number of neutralino, N in the sun (or in the earth) is given by

$$\dot{N} = C - C_A N^2 \tag{2.1}$$

where C is the capture rate of neutralino by the sun or the earth and C_A is the total annihilation cross section times relative velocity per volume. From eq.(2.1), we find that the present annihilation rate is

$$\Gamma_A = \frac{1}{2} C_A N^2 = \frac{1}{2} C \tanh^2(\sqrt{C C_A}\, t_0) \tag{2.2}$$

where $t_0 \simeq 4.5$ Gyr is the age of the solar system. For $\sqrt{CC_A}\ t_0 \ll 1$, the annihilation rate is $\Gamma_A \approx \frac{1}{2} C^2 C_A t_0^2$ and less than its maximal value. But, for $\sqrt{CC_A}\ t_0 \gg 1$, the neutralino density reach equilibrium and the annihilation rate is $\Gamma_A \approx \frac{1}{2} C$. Therefore, when accretion is efficient, the annihilation rate depends on the capture rate C, but not on the annihilation cross section.

In turn, the capture rate C strongly depends on the elastic scattering cross section of neutralino with matter in the sun and the earth. The capture rate for the earth primarily depends on the spin-independent DM scattering cross section. For the capture rate in the sun, however, both spin-independent and spin-dependent scattering cross section can be important and the significance of each contribution depends on the specific SUSY scenarios.

In MSSM, t-channel Higgs boson and s-channel squark exchange processes contribute to the spin-independent (scalar) scattering between neutralino and quarks. In many case, dominant contribution to the scalar cross section comes from the Higgs exchange process, which increases for large $\tan\beta$ and small Higgs masses and also if neutralino is a mixed gaugino-Higgsino state. On the other hand, for the spin-dependent cross section, t-channel Z boson and s-channel squark exchange processes contribute. Usually Z exchange contribution dominates, which is sensitive to Higgsino components of LSP, but largely independent of $\tan\beta$. Note that if the Higgsino component of the LSP increases, then both the spin-independent and the spin-dependent scattering cross sections will be enhanced, as shown below in the nonuniversal Higgs mass parameter case.

The capture rate C also depends on the local density of neutralino, ρ_χ and the neutralino velocity dispersion in the halo, $\bar{v}$ etc. For our numerical calculation, we use the code DARKSUSY [24] and fix $\bar{v} = 270$ km/s. For the local density of neutralino we fix $\rho_\chi = 0.3\,\mathrm{GeV/cm^3}$ if $\Omega_\chi h^2 \geq 0.025$, while performing a rescaling of the density as $\rho_\chi \to \rho_\chi\ (\Omega_\chi h^2/0.025)$ if $\Omega_\chi h^2 < 0.025$. Here Ω_χ is the neutralino relic density in units of the critical density and h is the present Hubble constant in units of 100 km s^{-1} Mpc^{-1}.

3. Upward-going muon flux in SUSY models

3.1 mSUGRA

In mSUGRA model, we assume a universal SUSY breaking scalar mass m, a universal gaugino mass M and a universal trilinear coupling A at GUT scale $m_{\mathrm{GUT}} \simeq 2 \times 10^{16}$ GeV. We also require that electroweak symmetry break radiatively and then the Higgsino mass parameter μ is determined by the condition:

$$\mu^2 = \frac{m_{H_d}^2 - m_{H_u}^2 \tan^2\beta}{\tan^2\beta - 1} - \frac{1}{2} M_Z^2\,. \tag{3.1}$$

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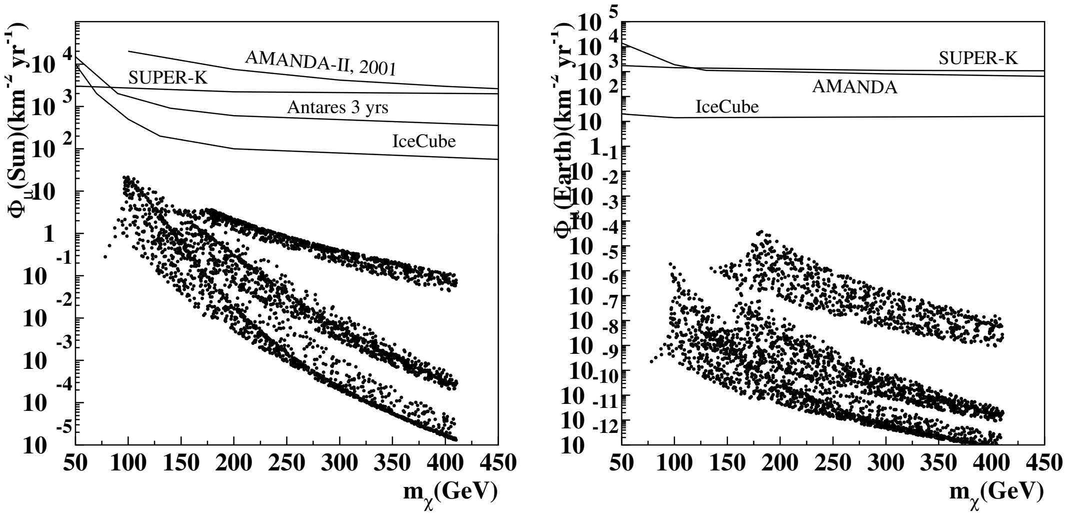

(a) Φ_μ (sun) (b) Φ_μ (earth)

Figure 1: The muon flux from the sun and the earth vs. m_χ in mSUGRA scenarios. The three branches correspond to $\tan\beta = 10, 35$ and 50 respectively (from bottom to top). The current upper limit from SUPER-K, AMANDA and the expected reaches of Antares and IceCube are also illustrated [25].

where $\tan\beta$ is the ratio of two Higgs vacuum expectation values and $m^2_{H_u}, m^2_{H_d}$ are the soft breaking Higgs masses-squared. With the above mSUGRA assumptions, $|\mu|$ is usually large so that the lightest neutralino is bino-like and the pseudo-scalar Higgs mass m_A is rather large.

In figures 1a and 1b, we show the allowed ranges of the upward-going muon fluxes from the sun and the earth respectively as functions of the LSP mass. The three branches correspond to $\tan\beta = 10, 35$ and 50 cases (from the bottom to the top), respectively. Here, we took $A = 0$ and $\mu > 0$ (motivated by the muon $(g-2)_\mu$ experiment) and varied m and M up to 1 TeV. We have imposed the experimental bounds for the Higgs and sparticle masses and for $b \to s\gamma$ branching ratio. We also required that the lightest neutralino is LSP. For opposite sign of μ, besides the muon $(g-2)_\mu$ data, the $b \to s\gamma$ constraint becomes more severe so that the maximal allowed values of both spin-dependent and independent DM scattering cross section are significantly reduced, compared the positive μ case. Therefore the muon flux becomes smaller. But we are interested in the possible maximal values, and we consider the positive μ case only in this work.

In our scan, the muon flux from the sun reaches up to $\sim 20\ km^{-2}yr^{-1}$ if the neutralino LSP is light enough ($m_\chi \sim 100\,\mathrm{GeV}$). In the small m_χ region, the neutralino density in the sun can reach (near) equilibrium so that the muon flux is more or less determined by the capture rate of neutralino by the sun. And in turn, the capture rate in the sun is determined primarily by the spin-dependent scattering cross section. (though the contribution from the spin-independent scattering cross section to the capture rate can be comparable to the one from the spin-dependent cross section for very large $\tan\beta$ cases) As we already mentioned in section 2, the spin-dependent scattering cross section is largely independent

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of $\tan\beta$. Therefore the upward-going muon flux from the sun in the small m_χ region gives similar values for the three choices of $\tan\beta$ values, as one can check from the figure.

In large m_χ region, the neutralino density is usually far from equilibrium since the elastic scattering cross section of the DM with ordinary matter in the sun becomes smaller, and the neutralino annihilation cross section becomes important for the prediction of the muon flux. An important process in this case is the neutralino pair annihilation into $b\bar{b}$ through s-channel pseudo-scalar Higgs exchange diagram, which is strongly enhanced for large $\tan\beta$ [26]. From the figure 1a, one can notice a clear dependence of the muon flux from the sun on $\tan\beta$ in the large m_χ region.

For the muon flux from the earth, the neutralino density is far less than the equilibrium values and both the capture rate and the annihilation rate are important for the calculation of the muon flux. The resulting flux is much below the one from the sun. The maximal value of the muon flux from the earth is about $3 \times 10^{-5}\ km^{-2}yr^{-1}$, which is far below the SUPER-K and IceCube sensitivity regions.

Note that there is no further constraint from the $B_s \to \mu^+\mu^-$ bound for the mSUGRA case, once we impose the constraints from the lower bounds for Higgs boson and SUSY particle masses and the $B \to X_s\gamma$ branching ratio, as discussed in ref. [7].

3.2 Non-universal Higgs model (NUHM)

In the previous subsection, we have shown that the mSUGRA assumption predicts the muon fluxes from the sun and the earth that are far below the sensitivity region of the current experiments. This is mainly because the lightest neutralino is bino-like and the pseudo-scalar Higgs mass is large in the scanned region of mSUGRA scenario. Larger muon flux from the sun and the earth can be obtained if we relax the universal boundary condition at GUT scale.

In this subsection we consider the non-universal Higgs model, in which the assumption of universal soft scalar masses are relaxed for soft Higgs masses, as follows:

$$m_{H_u}^2 = m^2(1 + \delta_{H_u})\,, \qquad m_{H_d}^2 = m^2(1 + \delta_{H_d})\,, \tag{3.2}$$

whereas other scalar masses still have a universal mass m at GUT scale. Here δ's are parameters with $\lesssim O(1)$.

As an optimal choice for enhancing the muon flux from the sun and the earth, we take the numerical values of δ's as $\delta_{H_d} = -1$ and $\delta_{H_u} = 1$. For the positive δ_{H_u}, μ becomes lower and the Higgsino component in the neutralino LSP increases so that $\sigma_{\chi p}$ is enhanced, as discussed in ref. [27, 28]. The change of $|\mu|$ also has an impact on the Higgs masses because

$$m_A^2 = m_{H_u}^2 + m_{H_d}^2 + 2\mu^2 \simeq m_{H_d}^2 + \mu^2 - \frac{M_Z^2}{2}$$

at weak scale. For the negative δ_{H_d}, m_A and m_H become further lower. As the result, both spin-independent scattering cross sections $\sigma_{\chi p}^{\rm scalar}$ and spin-dependent one $\sigma_{\chi p}^{\rm spin}$ are enhanced compared to mSUGRA case.

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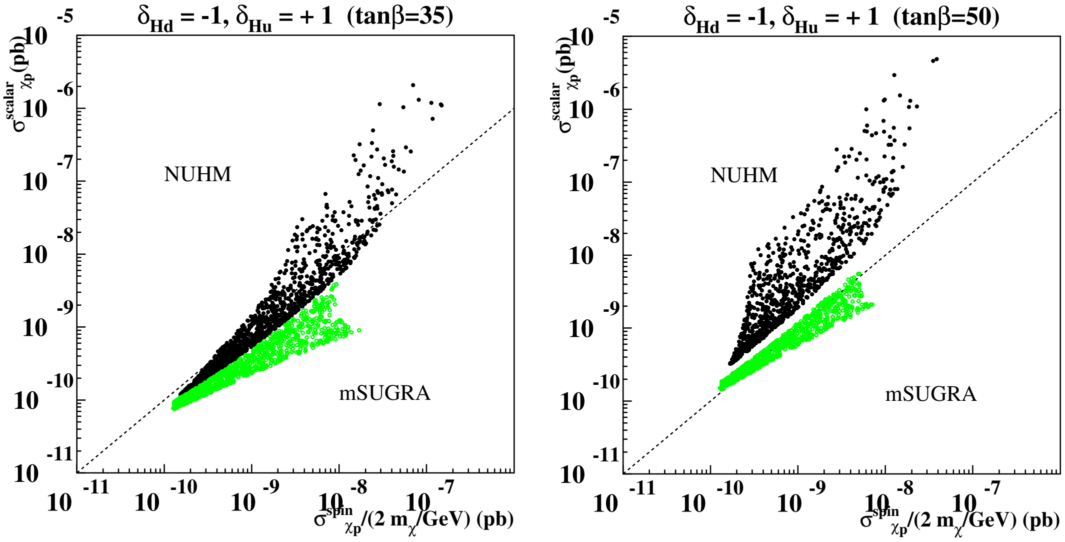
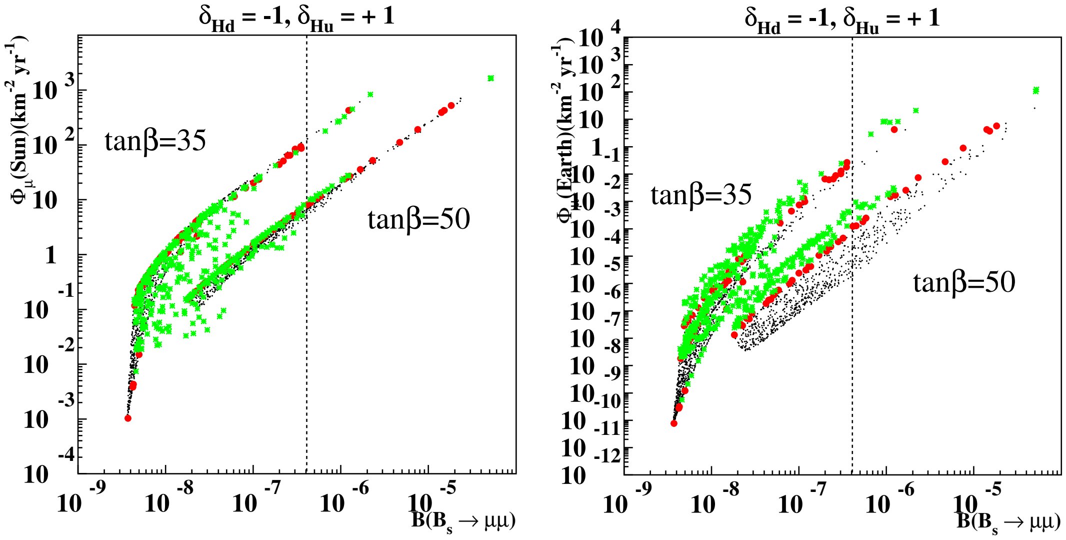


(*a*) $\tan\beta = 35$ (*b*) $\tan\beta = 50$

Figure 2: $\sigma^{\rm scalar}_{\chi p}$ vs. $\sigma^{\rm spin}_{\chi p}/(2m_\chi/{\rm GeV})$ in NUHM (black) and in mSUGRA (green) for (*a*) $\tan\beta = 35$ and (*b*) $\tan\beta = 50$.

In figure 2, we present $\sigma^{\rm scalar}_{\chi p}$ vs. $\sigma^{\rm spin}_{\chi p}/(2m_\chi/{\rm GeV})$ in the NUHM (black points) and mSUGRA scenario (green points) for (*a*) $\tan\beta = 35$ and (*b*) $\tan\beta = 50$ respectively. The dashed straight line in the figure indicates the region in which the two contributions to the capture rate are similar to each other. We observe that both $\sigma^{\rm spin}_{\chi p}$ and especially $\sigma^{\rm scalar}_{\chi p}$ in NUHM are enhanced a lot compared to mSUGRA scenario. An important point we notice from the figure is that $\sigma^{\rm scalar}_{\chi p}$ is usually (especially in the region of large cross section) larger than $\sigma^{\rm spin}_{\chi p}/(2m_\chi/{\rm GeV})$ in the NUHM case, while the opposite is true for the mSUGRA case. This is because the reduction of m_A values increase only $\sigma^{\rm scalar}_{\chi p}$ significantly (remember $\sigma^{\rm scalar}_{\chi p} \propto 1/m_A^4$ but $\sigma^{\rm spin}_{\chi p}$ is independent on m_A [4]), while the reduction of $|\mu|$ increases both $\sigma^{\rm scalar}_{\chi p}$ and $\sigma^{\rm spin}_{\chi p}$ due to the enhanced higgsino components of the lightest neutralino. This fact implies the capture rate for the sun in the NUHM is largely determined by the spin-independent scattering cross section rather than spin-dependent one, unlike the mSUGRA scenario. This is because the ratio of the contribution from spin-independent and spin-dependent cross section to the capture rate for the sun is approximately proportional to the ratio of $\sigma^{\rm scalar}_{\chi p}$ and $\sigma^{\rm spin}_{\chi p}/(2m_\chi/{\rm GeV})$.

As we have shown in the previous paper [7], the current experimental limit of $B(B_s \to \mu^+\mu^-)$ puts a strong constraint on the allowed range of the spin-independent cross section. Since the muon flux from the sun and the earth strongly depends on the spin-independent cross section, we naturally expect that the current limit of $B(B_s \to \mu^+\mu^-)$ play an important part in restricting the muon flux. This point can be observed clearly in figure 3, where we show explicitly the correlation between $B(B_s \to \mu^+\mu^-)$ and the muon flux from the sun (*a*) and the earth (*b*) in NUHM for $\tan\beta = 35$ and 50. Note that the $B_s \to \mu^+\mu^-$ is stronger for larger $\tan\beta$, and the resulting muon flux becomes smaller for the larger $\tan\beta$ case, like the spin independent DM scattering cross section [7].



(a) (b)

Figure 3: The muon fluxes from (a) the sun and (b) the earth vs. $B(B_s \to \mu^+\mu^-)$ in non-universal Higgs mass scenario. Here black dots for relic density $\Omega_\chi h^2 > 0.129$, red dots for $0.094 \le \Omega_\chi h^2 \le 0.129$ and green dots for $\Omega_\chi h^2 < 0.094$.

The enhancements of the neutralino DM scattering cross sections (both spin-dependent and spin-independent) lead to the substantial change of the muon flux both from the sun and the earth compared with the mSUGRA case. Figures 4a and 4b show the muon fluxes from the sun and the earth, respectively, in non-universal Higgs mass scenario with $\delta_{H_d} = -1, \delta_{H_u} = +1$ for $\tan\beta = 35$ case. Now the maximal values of the muon fluxes from the sun and the earth are $\sim 10^3$ (10) $km^{-2}yr^{-1}$, which is two (eight) orders of magnitude larger than the one for the mSUGRA case with $\tan\beta = 35$. In figure 4c and 4d, we show the muon flux from the sun and the earth, respectively, in non-universal Higgs mass scenario with $\delta_{H_d} = -1, \delta_{H_u} = +1$ for $\tan\beta = 50$. The red points (the open circles) are excluded by the current upper limit of $B(B_s \to \mu^+\mu^-)$.

3.3 D-brane model

Next, we consider a specific D brane model where the SM gauge groups and 3 generations live on different Dp branes [29]. In this model, scalar fermion masses are not completely universal and gaugino mass unification can be relaxed. Also the string scale is around 10^{12} GeV (the intermediate scale) rather than GUT scale.

Since there are now three moduli (T_i) and one dilaton superfields in this case, we use the following parametrization that is appropriate for several T_i moduli:

$$\begin{aligned} F^S &= \sqrt{3}\,(S + S^*)\; m_{3/2} \sin\theta, \\ F^i &= \sqrt{3}\,(T_i + T_i^*)\; m_{3/2} \cos\theta\;\Theta_i \end{aligned} \tag{3.3}$$

where θ and Θ_i $(i = 1, 2, 3)$ with $\sum_i |\Theta_i|^2 = 1$ parametrize the directions of the goldstinos

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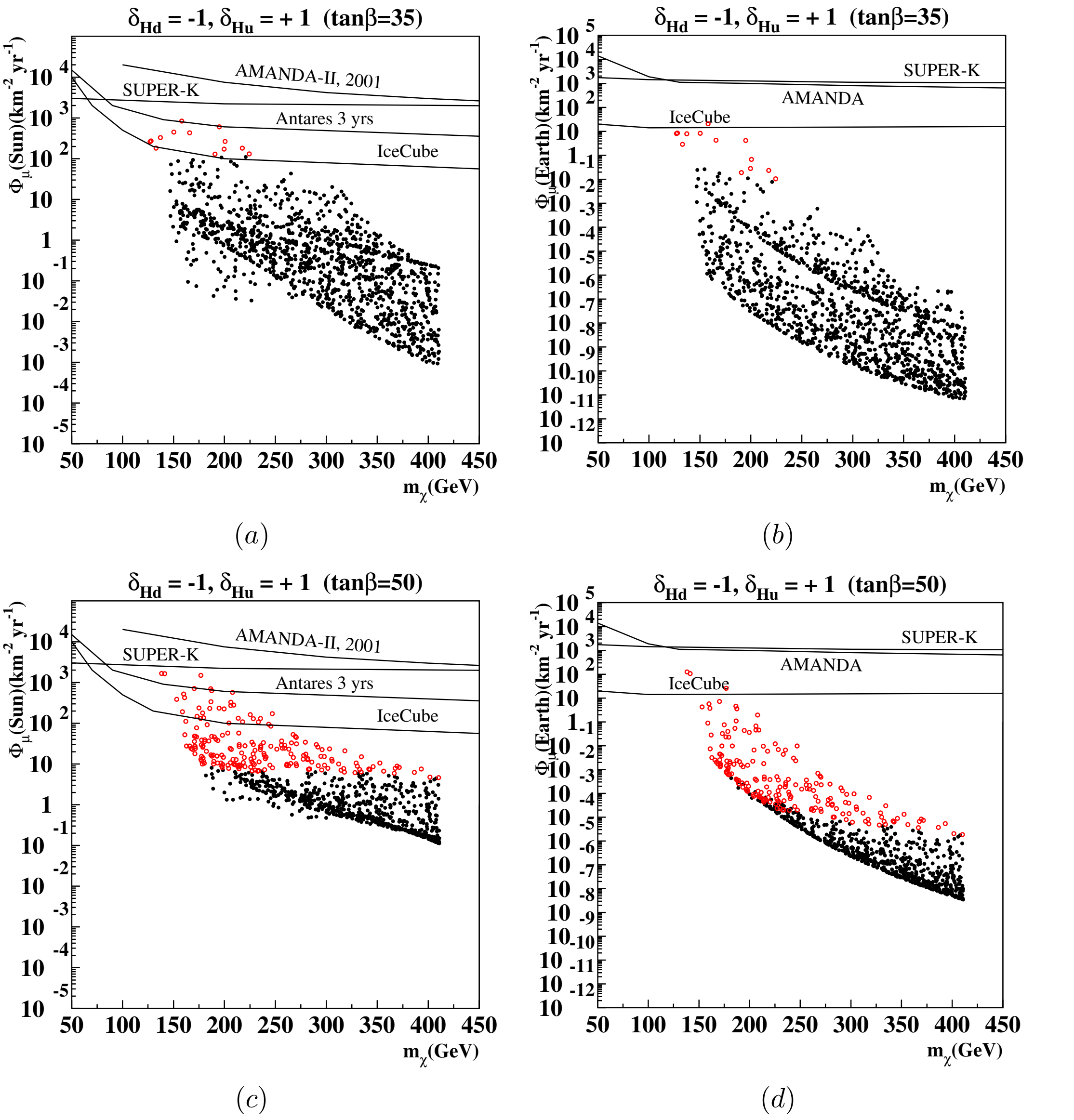


Figure 4: The muon flux from the sun and the earth vs. m_χ in Non-universal Higgs mass scenarios with $\delta_{H_d} = -1, \delta_{H_u} = +1$ and $\tan\beta = 35$ and $\tan\beta = 50$. The red points (the open circles) are excluded by the current upper limit of $B(B_s \to \mu^+\mu^-)$.

in the S, T_i field space. Then, the gaugino masses are given by

$$\begin{aligned}
M_3 &= \sqrt{3} m_{3/2} \sin\theta \,, \\
M_2 &= \sqrt{3} m_{3/2} \Theta_1 \cos\theta \,, \\
M_Y &= \sqrt{3} m_{3/2} \alpha_Y(M_I) \left(\frac{2\Theta_3 \cos\theta}{\alpha_1(M_I)} + \frac{\Theta_1 \cos\theta}{\alpha_2(M_I)} + \frac{2\sin\theta}{3\alpha_3(M_I)} \right) ,
\end{aligned} \tag{3.4}$$

where

$$\frac{1}{\alpha_Y(M_I)} = \frac{2}{\alpha_1(M_I)} + \frac{1}{\alpha_2(M_I)} + \frac{2}{3\alpha_3(M_I)} \,. \tag{3.5}$$

The string scale M_I is determined to be $M_I = 10^{12}\ (5\times 10^{14})\,\text{GeV}$ from the $\text{U}(1)_1$ gauge coupling $\alpha_1(M_I) = 0.1(1)$ [29]. Note that the gaugino masses are non universal in a natural way in this scenario, unlike other scenarios studied in the previous subsections.

The soft masses for the sfermions and Higgs fields are given by

$$\begin{aligned}
m_Q^2 &= m_{3/2}^2 \left[1 - \frac{3}{2}\left(1-\Theta_1^2\right)\cos^2\theta\right], \\
m_{u^c}^2 &= m_{3/2}^2 \left[1 - \frac{3}{2}\left(1-\Theta_3^2\right)\cos^2\theta\right], \\
m_{d^c}^2 &= m_{3/2}^2 \left[1 - \frac{3}{2}\left(1-\Theta_2^2\right)\ \cos^2\theta\ \right], \\
m_L^2 &= m_{3/2}^2 \left[1 - \frac{3}{2}\left(\sin^2\theta+\Theta_3^2\cos^2\theta\right)\right], \\
m_{e^c}^2 &= m_{3/2}^2 \left[1 - \frac{3}{2}\left(\sin^2\theta+\Theta_1^2\cos^2\theta\right)\right], \\
m_{H_2}^2 &= m_{3/2}^2 \ \left[1 - \frac{3}{2}\left(\sin^2\theta+\Theta_2^2\cos^2\theta\right)\right], \\
m_{H_1}^2 &= m_L^2\,.
\end{aligned} \tag{3.6}$$

Note that the scalar mass universality in the sfermion masses and Higgs masses is achieved when

$$\sin^2\theta = \frac{1}{4} \qquad \text{and} \qquad \Theta_i^2 = \frac{1}{3} \qquad \text{for } i = 1,2,3\,. \tag{3.7}$$

And in this case the gaugino masses becomes also universal, when we take only positive numbers for the solutions. For other choices of goldstino angles, the scalar and the gaugino masses become nonuniversal, and there could be larger or smaller flavor violation in the low energy processes as well as enhanced SUSY contributions to the a_μ^{SUSY}.

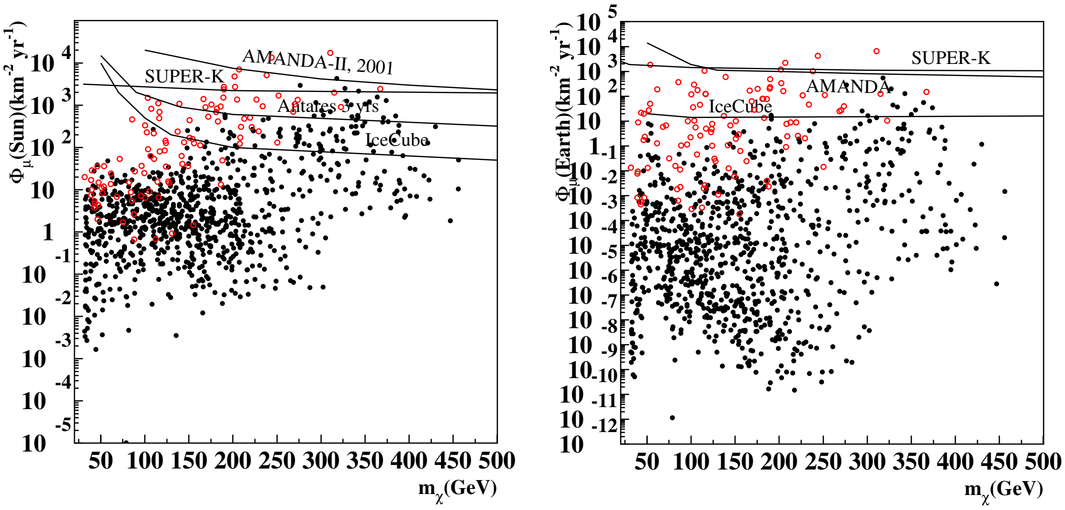
The trilinear couplings are given by

$$\begin{aligned}
A_u &= \frac{\sqrt{3}}{2} m_{3/2}\left[\left(\Theta_2-\Theta_1-\Theta_3\right)\cos\theta - \sin\theta\right], \\
A_d &= \frac{\sqrt{3}}{2} m_{3/2}\left[\left(\Theta_3-\Theta_1-\Theta_2\right)\cos\theta - \sin\theta\right], \\
A_e &= 0\,.
\end{aligned} \tag{3.8}$$

Therefore the D brane model considered in this work is specified by following six parameters:

$$m_{3/2}\,, \quad \tan\beta\,, \quad \theta\,, \quad \Theta_{i=1,2}\,, \quad \text{sign}(\mu)\,.$$

Due to the departure from the universality of scalar masses and the proportionality of trilinear couplings, the flavor violation could be different from the mSUGRA case. For example, it is possible to have smaller $b \to s$ transition due to the smaller $\tilde{t}_L - \tilde{t}_R$ mixing and larger stop masses in this $D-$brane scenarios, so that the $B(B_s \to \mu^+\mu^-)$ constraint can be relaxed. This can be seen in figure 5a and 5b, where the large flux signals would be excluded by Super-K and IceCube, but not by the $B(B_s \to \mu^+\mu^-)$ constraint. In this limited parameter space, one can have a large DM scattering cross section and the



(*a*) (*b*)

Figure 5: The muon flux from the sun and the earth vs. m_χ in a D-brane model with $\tan\beta = 50$. The red points (the open circles) are excluded by the current upper limit of $B(B_s \to \mu^+\mu^-)$.

upward-going muon flux without conflict with the $B_s \to \mu^+\mu^-$ branching ratio. Also there is no strong correlations among these observables. Therefore the indirect search for the DM annihilation is complementary to the $B_s \to \mu^+\mu^-$ branching ratio in the $D-$brane scenarios.

4. Conclusions

In this work, we considered the indirect detection of the DM through the upward-going muon flux from the DM annihilation at the core of the sun or the earth, along with the upper bound on the branching ratio for the $B_s \to \mu^+\mu^-$ decay, in some general supergravity scenarios where the upward-going muon flux could be enhanced very much compared to the mSUGRA case. In general supergravity scenario with non-universal Higgs masses, we found the following with the choice of model parameters considered in this paper:

- Both $\sigma_{\chi p}^{\rm spin}$ and $\sigma_{\chi p}^{\rm scalar}$ can be enhanced a lot compared to the mSUGRA scenario, but the enhancement in the spin-independent part is much greater.
- Therefore, contrary to the usual claim, the upward-going muon flux from the sun can be dominated by the spin-independent part $\sigma_{\chi p}^{\rm scalar}$ in the NUHM, rather than by the spin-dependent part $\sigma_{\chi p}^{\rm spin}$, as in the mSUGRA scenario [figure 2*a* and 2*b*].
- The current upper bound $B(B_s \to \mu^+\mu^-) < 4.1 \times 10^{-7}$ excludes a large parameter space where the muon fluxes could be enhanced otherwise, and the constraint is stronger for larger $\tan\beta$ [figure 3*a* and 3*b*].
- The upper bound on $B(B_s \to \mu^+\mu^-)$ becomes much stronger than the upper limits (the expected reach) on the muon flux from Super-K (IceCube) [figure 4*a*– 4*d*].

In the $D-$brane models with nonuniversal scalar fermion masses, the correlations between the muon flux and $B(B_s \to \mu^+\mu^-)$ becomes lost, and the upper bound on $B(B_s \to \mu^+\mu^-)$ is complementary to the upper bounds (the expected reach) on the muon fluxes from Super-K (IceCube). Our study shows that the muon flux originated from the DM annihilation in the sun could be in the range of a few $\times 10^3$ /km$^2\cdot$ yr.

Our study indicates that it is most important to include the $B_s \to \mu^+\mu^-$ branching ratio constraint when we study the direct and the indirect detections of the neutralino DM in general supergravity scenarios. The upper limit on the $B_s \to \mu^+\mu^-$ branching ratio excludes significant part of parameter space where the DM scattering cross section and the upward-going muon flux could be enhanced above/around the current experiments. Unless the chargino-stop contribution to $B_s \to \mu^+\mu^-$ is very small or there is fortuitous cancellation between the chargino-stop and the gluino-sbottom loop contributions, the spin-independent DM scattering cross section and the indirect detection rate through the upward-going muon flux are strongly constrained by the $B_s \to \mu^+\mu^-$ branching ratio. Since both the direct and the indirect detection rates are well below the current experiments in most supergravity model parameter space when the $B_s \to \mu^+\mu^-$ branching ratio constraint is imposed, it would be a great challenge for experimentalists to reach such sensitivity to have positive signals of the DM search.

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References

[1] WMAP collaboration, D.N. Spergel et al., *First year Wilkinson microwave anisotropy probe (WMAP) observations: determination of cosmological parameters*, *Astrophys. J. Suppl.* **148** (2003) 175 [astro-ph/0302209].

[2] H. Goldberg, *Constraint on the photino mass from cosmology*, *Phys. Rev. Lett.* **50** (1983) 1419;
J.R. Ellis, J.S. Hagelin, D.V. Nanopoulos, K.A. Olive and M. Srednicki, *Supersymmetric relics from the big bang*, *Nucl. Phys.* **B 238** (1984) 453.

[3] M.W. Goodman and E. Witten, *Detectability of certain dark-matter candidates*, *Phys. Rev.* **D 31** (1985) 3059.

[4] For a review, see G. Jungman, M. Kamionkowski and K. Griest, *Supersymmetric dark matter*, *Phys. Rept.* **267** (1996) 195 [hep-ph/9506380].

[5] DAMA collaboration, R. Bernabei et al., *Search for WIMP annual modulation signature: results from DAMA/NaI-3 and DAMA/NaI-4 and the global combined analysis*, *Phys. Lett.* **B 480** (2000) 23.

[6] CDMS collaboration, D.S. Akerib et al., *First results from the cryogenic dark matter search in the Soudan underground lab*, *Phys. Rev. Lett.* **93** (2004) 211301 [astro-ph/0405033].

[7] S. Baek, Y.G. Kim and P. Ko, *Neutralino dark matter scattering and $B_s \to \mu^+\mu^-$ in SUSY models*, *JHEP* **02** (2005) 067 [hep-ph/0406033].

[8] K.S. Babu and C.F. Kolda, *Higgs-mediated $B^0 \to \mu^+\mu^-$ in minimal supersymmetry*, *Phys. Rev. Lett.* **84** (2000) 228 [hep-ph/9909476];
G. Isidori and A. Retico, *Scalar flavour-changing neutral currents in the large-$\tan\beta$ limit*, *JHEP* **11** (2001) 001 [hep-ph/0110121];
A. Dedes, H.K. Dreiner and U. Nierste, *Correlation of $B_s \to \mu^+\mu^-$ and $(g-2)_\mu$ in minimal supergravity*, *Phys. Rev. Lett.* **87** (2001) 251804 [hep-ph/0108037];
G. Isidori and A. Retico, *$B_{s,d} \to \ell^+\ell^-$ and $K_L \to \ell^+\ell^-$ in SUSY models with non-minimal sources of flavour mixing*, *JHEP* **09** (2002) 063 [hep-ph/0208159];
A. Dedes and A. Pilaftsis, *Resummed effective lagrangian for Higgs-mediated fcnc interactions in the CP-violating MSSM*, *Phys. Rev.* **D 67** (2003) 015012 [hep-ph/0209306].

[9] G.L. Kane, C. Kolda and J.E. Lennon, *$B_s \to \mu\mu$ as a probe of $\tan\beta$ at the Tevatron*, hep-ph/0310042.

[10] S. Baek, P. Ko and W.Y. Song, *Implications on SUSY breaking mediation mechanisms from observing $B_s \to \mu^+\mu^-$ and the muon $(g-2)$*, *Phys. Rev. Lett.* **89** (2002) 271801 [hep-ph/0205259].

[11] S. Baek, P. Ko and W.Y. Song, *SUSY breaking mediation mechanisms and $(g-2)_\mu$, $B \to X_s\ell^+\ell^-$ and $B_s \to \mu^+\mu^-$*, *JHEP* **03** (2003) 054 [hep-ph/0208112].

[12] J.R. Ellis, K.A. Olive and V.C. Spanos, *On the interpretation of $B_s \to \mu^+\mu^-$ in the CMSSM*, *Phys. Lett.* **B 624** (2005) 47 [hep-ph/0504196].

[13] S. Baek, D.G. Cerdeno, Y.G. Kim, P. Ko and C. Muñoz, *Direct detection of neutralino dark matter in supergravity*, *JHEP* **06** (2005) 017 [hep-ph/0505019].

[14] J. Silk, K.A. Olive and M. Srednicki, *The photino, the sun and high-energy neutrinos*, *Phys. Rev. Lett.* **55** (1985) 257;
K. Freese, *Can scalar neutrinos or massive dirac neutrinos be the missing mass?*, *Phys. Lett.* **B 167** (1986) 295;
L.M. Krauss, M. Srednicki and F. Wilczek, *Solar system constraints and signatures for dark matter candidates*, *Phys. Rev.* **D 33** (1986) 2079.

[15] BAKSAN collaboration, M.M. Boliev et al., *Search for supersymmetric dark matter with Baksan Underground Telescope*, *Nucl. Phys.* **48** *(Proc. Suppl.)* (1996) 83

[16] MACRO collaboration, M. Ambrosio et al., *Limits on dark matter WIMPs using upward-going muons in the macro detector*, *Phys. Rev.* **D 60** (1999) 082002 [hep-ex/9812020].

[17] Super-Kamiokande collaboration, S. Desai et al., *Search for dark matter WIMPs using upward through-going muons in super-Kamiokande*, *Phys. Rev.* **D 70** (2004) 083523 [hep-ex/0404025], erratum *ibid.* **D70** (2004) 109901.

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[18] AMANDA collaboration, J. Ahrens et al., *Limits to the muon flux from WIMP annihilation in the center of the earth with the amanda detector*, *Phys. Rev.* **D 66** (2002) 032006 [astro-ph/0202370].

[19] ANTARES collaboration, L. Moscoso, *Neutrino telescopes under the ocean: the case for antares*, *Nucl. Phys.* **77** *(Proc. Suppl.)* (1999) 492 [hep-ex/9809020].

[20] The IceCube collaboration, J. Ahrens et al., *Icecube: the next generation neutrino telescope at the south pole*, *Nucl. Phys.* **118** *(Proc. Suppl.)* (2003) 388 [astro-ph/0209556].

[21] e.g. Anassontzis et al., *Status of nestor, a deep sea neutrino telescope in the mediterranean*, *Nucl. Phys.* **66** *(Proc. Suppl.)* (1998) 247.

[22] CDF collaboration, D. Acosta et al., *Search for $B_s^0 \to \mu^+\mu^-$ and $B_d^0 \to \mu^+\mu^-$ decays in $p\bar{p}$ collisions at $\sqrt{s} = 1.9\,6$-TeV*, *Phys. Rev. Lett.* **93** (2004) 032001 [hep-ex/0403032].

[23] D0 collaboration, V.M. Abazov et al., *A search for the flavor-changing neutral current decay $B_s^0 \to \mu^+\mu^-$ in $p\bar{p}$ collisions at $\sqrt{s} = 1.9\,6$-TeV with the D0 detector*, *Phys. Rev. Lett.* **94** (2005) 071802 [hep-ex/0410039].

[24] P. Gondolo, J. Edsjö, L. Bergstrom, P. Ullio and E.A. Baltz, *Darksusy: a numerical package for dark matter calculations in the MSSM*, astro-ph/0012234.

[25] J. Edsjö, *Indirect detection of WIMP*, Proc. Int. Conf. Neutrino-2004, Paris.

[26] V. Bertin, E. Nezri and J. Orloff, *Neutrino indirect detection of Neutralino dark matter in the CMSSM*, *Eur. Phys. J.* **C 26** (2002) 111.

[27] C. Muñoz, *Dark matter detection in the light of recent experimental results*, *Int. J. Mod. Phys.* **A 19** (2004) 3093 [hep-ph/0309346]; *Theoretical predictions for the direct detection of supersymmetric dark matter*, hep-ph/0312321.

[28] D.G. Cerdeno, E. Gabrielli, M.E. Gomez and C. Muñoz, *Neutralino nucleon cross section and charge and colour breaking constraints*, *JHEP* **06** (2003) 030 [hep-ph/0304115].

[29] D.G. Cerdeno et al., *Determination of the string scale in D-brane scenarios and dark matter implications*, *Nucl. Phys.* **B 603** (2001) 231 [hep-ph/0102270].

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